

# A Pareto-metaheuristic for a bi-objective winner determination problem in a combinatorial reverse auction

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## Abstract

The bi-objective winner determination problem (2WDP-SC) of a combinatorial procurement auction for transport contracts comes up to a multi-criteria set covering problem. We are given a set  $B$  of bundle bids. A bundle bid  $b \in B$  consists of a bidding carrier  $c_b$ , a bid price  $p_b$ , and a set  $\tau_b$  of transport contracts which is a subset of the set  $T$  of tendered transport contracts. Additionally, the transport quality  $q_{t,c_b}$  is given which is expected to be realized when a transport contract  $t$  is executed by a carrier  $c_b$ . The task of the auctioneer is to find a set  $X$  of winning bids ( $X \subseteq B$ ), such that each transport contract is part of at least one winning bid, the total procurement costs are minimized, and the total transport quality is maximized. This article presents a metaheuristic approach for the 2WDP-SC which integrates the greedy randomized adaptive search procedure, large neighborhood search, and self-adaptive parameter setting in order to find a competitive set of non-dominated solutions. The procedure outperforms existing heuristics. Computational experiments performed on a set of benchmark instances show that, for small instances, the presented procedure is the sole approach that succeeds to find all Pareto-optimal solutions. For each of the large benchmark instances, according to common multi-criteria quality indicators of the literature, it attains new best-known solution sets.

**Keywords:** Pareto optimization, multi-criteria winner determination, combinatorial auction, GRASP, LNS

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## 1. Introduction and literature review

Combinatorial auctions are applied when bidders are interested in multiple heterogeneous items and when the bidders' valuations of these items are non-additive. This is for example the case with the procurement of transport services which often are highly interdependent. We focus on these kinds of items in the following. In a combinatorial transport auction, a shipper wants to procure transport services from many freight carriers. Items of a transport auction are denoted as transport contracts. Such a contract is a framework agreement with a duration of about one to three years, that defines an origin location and a destination location between which a certain volume of goods has to be regularly carried (usually on the road) while a specified service level has to be satisfied.

Combinatorial transport auctions allow freight carriers (bidders) to submit bundle bids. A *bundle bid* is an all-or-nothing bid on any subset of the set of tendered transport contracts. In particular, a freight carrier can bid on combinations of transport contracts that exhibit strong synergies ([1], [2], [3]). With this, the shipper strives to reduce his or her total transport costs.

Real-world applications of combinatorial auctions for the procurement of transport service are described by Ledyard et al. [4], Elmaghraby and Keskinocak [5], for example. Caplice and Sheffi [6, 7] discuss real-world issues of combinatorial transport auctions and report, among other things, that practical transport auctions studied handle an average annual procurement volume of 150 million US-dollar. The whole auction process is complex and can last a few months [6].

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After bidding is completed, the shipper (auctioneer) has to decide which of the received bundle bids should be accepted as winning bids. This problem is known as the winner determination problem which is usually modeled as a combinatorial optimization problem (for a review see [8]). For combinatorial auctions which are used for selling items, the set packing problem is used to maximize the total revenue (compare [9, 10], for a review see [11]). Conversely, the winner determination problem of combinatorial procurement auctions like transport auctions are often modeled based on the set covering problem or the set partitioning problem and the total procurement costs are minimized.

In practice, shippers usually also want to ensure or improve service quality of the procured transport contracts ('transport quality') and therefore do not exploit their full potential for cost savings [3]. Models of winner determination problems of combinatorial auctions that try to integrate quality aspects in the decision making process are described in [6], [7], [12], [13]. Primarily, these approaches try to integrate quality aspects as some kind of side constraint or they use penalty costs to disadvantage low quality carriers or bundle bids, respectively. However, this requires preference information of the shipper with respect to the desired trade-off between transport costs and transport quality. As Caplice and Sheffi [6] state, identifying the desired trade-off is one of the most challenging tasks in the procurement of transport contracts for a shipper. Therefore, Buer and Pankratz [14] introduced an additional, second objective function for maximizing the transport quality within a winner determination problem. The resulting bi-objective model, denoted as 2WDP-SC, seems helpful, if the desired trade-off between transport costs and transport quality is a priori unknown to the shipper.

This paper presents a new heuristic for a bi-objective winner determination problem. The presented heuristic outperforms previous methods for that optimization problem [14, 15, 16]. The article is organized as follows. Section 2 introduces the studied bi-objective winner determination problem. To solve it, we present a new Pareto metaheuristic called PNS (Section 3). The performance of PNS is evaluated by means of a benchmark study (Section 4) whose results are discussed in Section 5. Section 6 summarizes the findings.

## 2. The bi-objective winner determination problem

The bi-objective winner determination problem of a combinatorial transport procurement auction based on a set covering formulation (2WDP-SC) has been introduced by Buer and Pankratz [14]. We are given a set  $T$  of transport contracts offered by a single shipper (decision maker) and a set  $B$  of bundle bids which have been submitted by a set  $C$  of carriers. A bundle bid  $b \in B$  is composed of a carrier  $c_b \in C$ , a bid price  $p_b \in \mathbb{R}^+$ , and a subset  $\tau_b$  of the offered transport contracts  $T$ . With the bundle bid  $b$ , the carrier  $c_b \in C$  expresses the intention to execute the set of transport contracts  $\tau_b \subseteq T$ , if he gets paid the price  $p_b$  by the shipper. Let  $a_{tb} = 1$  if  $t \in \tau_b$  and  $a_{tb} = 0$  otherwise ( $\forall t \in T, \forall b \in B$ ). If  $a_{tb} = 1$ , we say  $b$  covers  $t$ . Furthermore, we are given parameters  $q_{t,c_b} \in \mathbb{N}$  ( $\forall t \in T, c \in C$ ) which indicate the achieved transport quality if transport contract  $t$  is executed by carrier  $c$  who submitted bundle bid  $b \in B$ . The shipper prefers higher values of  $q_{t,c_b}$ .

The optimization task of the shipper is to determine a set of winning bids  $X$  ( $X \subseteq B$ ). The binary decision variable  $x_b$  indicates, whether bundle bid  $b \in B$  is accepted as winning bid ( $x_b = 1 \Leftrightarrow b \in X$ ) or not. The 2WDP-SC asks for the set of winning bids  $X$  that covers all transport contracts  $T$  and at the same time strives to do both, to minimize the total procurement costs and to maximize the total transport quality. The 2WDP-SC is defined by the expressions (1) – (4).

$$\min f_1(X) = \sum_{b \in B} p_b \cdot x_b, \quad (1)$$

$$\min f_2(X) = (-1) \sum_{t \in T} \max_{b \in B} \{q_{t,c_b} \cdot a_{tb} \cdot x_b\}, \quad (2)$$

$$\text{s. t.} \quad \sum_{b \in B} a_{tb} \cdot x_b \geq 1, \quad \forall t \in T, \quad (3)$$

$$x_b \in \{0, 1\}, \quad \forall b \in B. \quad (4)$$

Objective function  $f_1$  (1) minimizes the total procurement costs of the shipper. That is, the sum of the prices of the winning bids. Objective function  $f_2$  (2) maximizes the total transport quality of the procured transport contracts. For ease of notation used later, we minimize the negative total transport quality to obtain a pure minimization problem.

Constraint set (3) guarantees, that each transport contract is covered by *at least one* winning bid. Finally, expression (4) ensures, that each bundle bid is an all-or-nothing bid, that is, partial acceptance of a bundle bid is prohibited.

The formulation of the objective function  $f_2$  is influenced by the set covering inequality (3). Because of (3), a transport contract  $t$  may be covered by multiple winning bids although it must be executed only once (this is possible due to the free disposal assumption). Therefore, the maximum function in  $f_2$  makes sure, that for each transport contract  $t$  only the highest transport quality value  $q_{t,c_b}$  for the given the set of winning bids is summed up once. Note, that using the set partitioning equality with a strict equal sign instead of (3) would avoid this issue – however, this would complicate finding a feasible solution and most likely lead to higher total procurement costs  $f_1$  which is unwanted by the shipper (using set covering or set partitioning variant in this context is discussed in more detail by Buer and Pankratz [15, p. 195f]).

The expressions (1), (3), and (4) define the well-known NP-hard set covering problem [17]. If a single objective decision problem with  $f_k, k = 1$  is NP-complete, then the corresponding multi objective decision variant with  $f_k, k > 1$  is also NP-complete [18]. Therefore, the 2WDP-SC is NP-hard.

Finally, we introduce the notation of solution dominance. Let  $k$  be the number of objective functions of a minimization problem and let  $X^1, X^2$  be two feasible solutions.  $X^1$  *weakly dominates*  $X^2$ , written  $X^1 \leq X^2$ , if  $f_i(X^1) \leq f_i(X^2), i = 1, \dots, k$ .  $X^1$  *dominates*  $X^2$ , written  $X^1 < X^2$ , if  $f_i(X^1) \leq f_i(X^2), i = 1, \dots, k$  and  $f_i(X^1) < f_i(X^2)$  holds at least for one  $k$ . An *approximation set* is a set of feasible solutions which do not  $<$ -dominate each other. The approximation set which contains those feasible solutions which are not weakly dominated by any other feasible solution is called Pareto-optimal set.

### 3. A Pareto metaheuristic based on GRASP and adaptive LNS

The developed metaheuristic procedure is denoted as Pareto neighborhood search (PNS). It integrates search techniques known from the metaheuristics greedy randomized adaptive search procedures (GRASP) and large neighborhood search (LNS). With respect to the multicriteria situation, the search uses dominance-based and criterion-individual search techniques (cf. Talbi [19, p. 323ff]). Dominance-based search means that values of both objective functions are used to control the search process, while criterion individual search means that only a single objective criterion is used and the other is temporarily neglected. An overview of PNS is given in Alg. 1.

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#### Algorithm 1: Pareto neighborhood search (PNS)

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**Input:** problem data:  $B$ ; parameters:  $r, d^{max}, ds$

**Output:** approximation set  $A$

$A \leftarrow \text{dominanceBasedConstruction}(B, r, d^{max})$

$A \leftarrow \text{localSearch}(B, ds, A)$

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#### 3.1. Construction Phase (DRC)

A set of non-dominated solutions is constructed with a method called *dominance-based randomized construction* (DRC, cf. Alg. 2). DRC is a multi start procedure that iteratively constructs feasible solutions to obtain a good approximation set. The termination of the multi start procedure is controlled by the parameter  $d^{max} \in \mathbb{N}$ . That is, DRC terminates if  $d^{max}$  solutions are constructed successional without finding a new non-dominated solution.

A single feasible solution is actually constructed by adding bundle bids successively to an infeasible solution  $X$ . The bid to be added next is determined via a *two-stage candidate bid selection* procedure.

In the *first stage*, a set of candidate bids, also denoted as candidate list  $CL \subset B \setminus X$ , is determined. Bids in  $CL$  are potential candidates to be added to the current solution  $X$ . Therefore, we use the vector-valued greedy function  $g(b, X) = (P(b, X), Q(b, X))$  to rate every bundle bid which was also used by Buer and Pankratz [15]. The elements of the candidate list  $CL$  are those bundle bids which are not dominated by other bundle bids with respect to the valuation of  $g(b, X)$ .

The first rating function  $P(b, X)$  measures the ability of a bundle bid  $b \notin X$  to make  $X$  a feasible solution and to improve  $f_1(X)$ . Lower values of  $P(b, X)$  are considered as better.  $P(b, X)$  calculates the average additional costs for

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**Algorithm 2:** DRC

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**Input:** set of bundle bids  $B$ , no. sectors  $r$ ,  $d^{max}$

**Output:** approximation set  $A$

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$d \leftarrow 1$

**repeat**

// restart loop

$k \leftarrow 0$

$X \leftarrow \emptyset$

$B' \leftarrow B$

**while**  $X$  infeasible **do**

$CL \leftarrow \emptyset$

R1      **foreach**  $b \in B \setminus X$  **do**

**if**  $g(b, X) = \infty$  **then**

$B \leftarrow B \setminus \{b\}$

**else**

R2           $CL \leftarrow CL \uplus \{b\}$

**end**

**end**

$b' \leftarrow \text{sectorCandSelection}(CL, k, r)$

$X \leftarrow X \cup \{b'\}$

$k \leftarrow k + 1$

**end**

**if** ( $\nexists X' \in A | X' < X$ ) **then**

$A \leftarrow A \uplus \{X\}$

$d \leftarrow 1$

**else**

$d \leftarrow d + 1$

**end**

$B \leftarrow B'$

**until**  $d = d^{max}$

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those contracts in  $b$  which are not yet covered by  $X$  (cf. Chvátal [20]). Let  $\tau(X)$  denote the set of contracts covered by  $X$ , i.e.  $\tau(X) = \bigcup_{b \in X} \tau(b)$ . If all transport contracts  $\tau(b)$  of a bundle bid  $b$  are already covered by  $X$ , then  $b$  cannot contribute to reach feasibility of  $X$  and therefore  $P(b, X)$  is set to  $+\infty$ .  $P(b, X)$  is defined according to (5).

$$P(b, X) = \begin{cases} \frac{p_b}{|\tau(b) \setminus \tau(X)|} & \text{if } \tau(b) \setminus \tau(X) \neq \emptyset, \\ +\infty & \text{otherwise.} \end{cases} \quad (5)$$

The second rating function  $Q(b, X)$  measures the ability of a bundle bid  $b \notin X$  to improve  $f_2(X)$ . By accepting an additional bundle bid  $b$  as winning bid the transport quality  $f_2$  either remains stationary or increases, i.e.,  $\Delta f_2(X) = f_2(X \cup \{b\}) - f_2(X) \geq 0$ . In contrast to  $f_1$ , the value of  $f_2$  cannot worsen by accepting an additional bid. The increment in transport quality  $\Delta f_2(X)$  is divided by the total number of contracts covered by each individual bid in  $X \cup \{b\}$ , that is  $\sum_{b' \in X \cup \{b\}} |\tau(b')|$ . By this, covering a contract by several bids is penalized. Finally, this value is multiplied by  $-1$ , so that smaller values of  $Q(b, X)$  represent better bids (in consistence with  $P(b, X)$ ). If  $\Delta f_2(X) = 0$ ,  $b$  does not improve  $f_2$  and  $Q(b, X)$  assigns a rating of  $+\infty$ .  $Q(b, X)$  is defined according to (6).

$$Q(b, X) = \begin{cases} -\frac{\Delta f_2(X)}{\sum_{b' \in X \cup \{b\}} |\tau(b')|} & \text{if } \Delta f_2(X) > 0, \\ +\infty & \text{otherwise.} \end{cases} \quad (6)$$

By means of the vector-valued rating function  $\mathbf{g}(b, X)$  the candidate list is constructed during the foreach-loop of DRC (cf. remark R1 of Alg. 2). As long as the constructed solution  $X$  is infeasible, the following steps are performed: Each

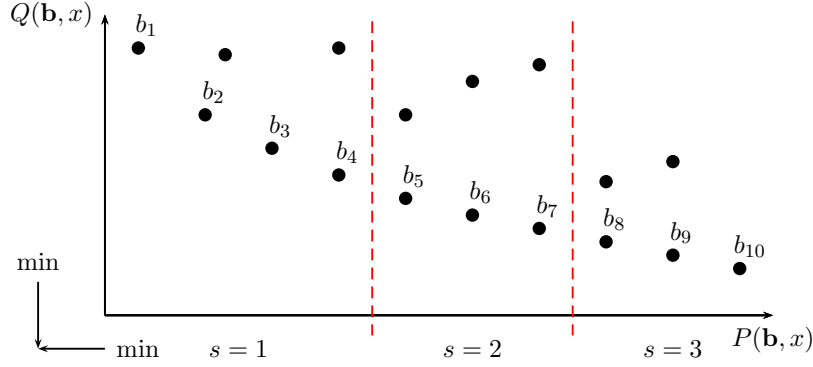


Figure 1: Organization of candidate list

bundle bid  $b \in B \setminus X$  is rated according to  $\mathbf{g}(b, X)$ . If the rated bundle bid  $b$  is not dominated by any of the bundle bids in  $CL$ , then  $b$  is added to  $CL$  and those bundle bids in  $CL$  which are dominated by  $b$  are removed. This is symbolized by the operator  $\uplus$  (cf. remark R2 of Alg. 2). On the other hand, if  $\mathbf{g}(b, X) = (+\infty, +\infty)$  then  $b$  is not able to contribute to the constructed solution  $X$  and can be disregarded in future ratings of the same solution.

After all bundle bids in  $B \setminus X$  have been rated and the set of candidate bids  $CL$  is available, a bundle bid has to be selected from  $CL$  and added to  $X$  at random. This is done in the *second stage* of the two-stage candidate bid selection procedure.

In the second stage, the procedure *sectorCandSelect* (cf. Alg. 3) selects a particular bundle bid  $b \in CL$  that should be added to the infeasible solution  $X$ . The procedure *sectorCandSelect* requires as input the candidate list  $CL \subseteq B$ , the number of up to now constructed solutions  $k \in \mathbb{N}$ , and the external parameter  $r \in \mathbb{N}$ .

First, the bundle bids of the given candidate list  $CL$  are partitioned into  $r$  subsets  $CL_s \subset CL$  which are denoted as *sectors*. Second, depending on the number of up to now constructed solutions  $k$  a sector  $CL_s$  is selected from which a bundle bid  $b$  is randomly chosen with probability  $1/|CL_s|$ .

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**Algorithm 3:** sectorCandSelection

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**Input:** candidate list  $CL$ , mult start counter  $k$ , no. sectors  $r$   
**Output:** a selected bundle bid  $b, b \in CL$

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 $n \leftarrow |CL|$ 
if  $r > n$  then  $r \leftarrow n$ 
 $m_j \leftarrow \lfloor n/r \rfloor$  // cardinality of  $CL_j, 1 < j < r$ 
 $m_1 \leftarrow n - m_j \cdot (r - 1)$  // cardinality of  $CL_1$ 
sort elements of  $CL$  in increasing order of  $P(b, X)$ 
6  $s \leftarrow k \bmod r + 1$  // determine sector
if  $s = 1$  then
|  $CL_s \leftarrow CL[1, m_1]$ 
else
|  $CL_s \leftarrow CL[m_1 + m_j \cdot (s - 2) + 1, m_1 + m_j \cdot (s - 1)]$ 
end
select a bid  $b \in CL_s$  with probability  $1/|CL_s|$ 
return  $b$ 

```

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Each sector  $CL_s$  should contain an equal number of bundle bids. If an equal division of bids to sectors is not possible ( $|CL| \bmod r > 0$ ), then the remaining bids are assigned to the first sector  $CL_1$ . Therefore,  $|CL_1| \geq |CL_2| = \dots = |CL_r|$ . In the example of Fig. 1, the candidate list  $CL$  is made up of ten non-dominated bundle bids  $b_1, \dots, b_{10}$ . These are divided into  $r = 3$  sectors. Sector  $CL_1$  contains four bids and sectors  $CL_2$  and  $CL_3$  contain three bids, respectively.

From the sector  $CL_s$ , a bundle bid is chosen randomly with equal probability. The sector  $CL_s$  is determined

depending on the number of constructed solutions  $k$  so far (cf. Line 6 of Alg. 3). To construct the first solution ( $k = 0$ ), all bundle bids are drawn from the first sector  $CL_1$ . For the second solution ( $k = 1$ ), all bundle bids are drawn from sector  $CL_2$  and so forth.

The idea of segmenting the bundle bids into sectors, is to steer the search process into certain dimensions of the bicriteria objective space. The options to choose a bundle bid in each construction step are reduced and the pressure to steer into a certain part of the objective space is increased. Without any segmentation ( $r = 1$ ), decisions made in the later stage of constructing a solution might conflict previous decisions. For example, first some bundle bids are chosen which might result in a good solution with respect to the first objective, later some other bundle bids are chose which are in favor of the second objective function; sometimes this will lead to good compromise solution but sometimes the solution quality will be poor in both objective functions.

### 3.2. Improvement phase

The improvement phase (cf. Alg. 4) is inspired by the metaheuristic large neighborhood search. Basically, solutions from the approximation set  $A$  are destroyed randomly according to a destroy rate and after that rebuilt by means of a greedy single-criterion method. The actual destroy rate and the actual choice of one of the two greedy rating functions  $P(b, X)$  and  $Q(b, X)$  are decided during the improvement phase by setting parameters self-adaptive.

The main criterion for the self-adaptive parameter setting, that is the choice of a destroy rate as well as a greedy rating function, is the number of failed attempts to improve  $X \in A$ . This measure for 'success' of a certain destroy rate and a certain greedy rating function is tracked on a local level for each solution and not on a global level for the entire approximation set. With this focus on individual solutions in  $A$ , the improvement phase is able to better account for structural differences between non-dominated solutions. Structural differences on the decision space level may occur for solutions that lie in very different areas of the objective space but are nevertheless Pareto optimal. For example, a solution  $X$  with a high value for  $f_1(X)$  and a low value for  $f_2(X)$  versus a solution  $X'$  with a low value for  $f_1(X')$  and a high value for  $f_2(X')$ .

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#### Algorithm 4: Improvement phase

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**Input:** approximation set  $A$ ,  $ds$

---

**repeat**

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R1  select and remove the first solution  $X$  of  $A$ 
    reinsert  $X$  at back of  $A$ 
     $X^d \leftarrow \text{destroySol}(X, \text{fail}[X].P, \text{fail}[X].Q, ds)$ 
     $X^r \leftarrow \text{repairSol}(X^d, \text{fail}[X].P, \text{fail}[X].Q)$ 
    if  $\nexists X^a \in A \mid X^a \leq X^r$  then
R2  |    $A \leftarrow A \cup \{X^r\}$                                      // insert at back of  $A$ 
    |    $\text{fail}[X^r].P \leftarrow 0$ 
    |    $\text{fail}[X^r].Q \leftarrow 0$ 
    else if  $\text{fail}[X].P < \text{fail}[X].Q$  then
    |    $\text{fail}[X].P \leftarrow \text{fail}[X].P + 1$ 
    else
    |    $\text{fail}[X].Q \leftarrow \text{fail}[X].Q + 1$ 
    end
until time limit reached
return  $A$ 

```

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The improvement phase (cf. Alg. 4) requires as input an approximation set and a destroy strategy. A destroy strategy  $ds$  is a sequence  $\langle ds_1, \dots, ds_n \rangle$  of destroy rates ( $\forall ds_i > 0$ ). The notations  $\text{fail}[X].P$  and  $\text{fail}[X].Q$  in Alg. 4 denote the number of failed attempts to improve the solution  $X$  with the greedy rating function  $P(., X)$  and  $Q(., X)$ , respectively. Furthermore, the approximation set  $A$  is implemented as a first-in, first-out list structure. To remove a solution from  $A$ , the solution on the first position is chosen (cf. remark R1); a new non dominated solution  $X$  is inserted at the back of  $A$ . At the same time, solutions in  $A$  which are dominated by  $X$  are deleted (cf. remark R2). With

this first-in, first-out approximation list, the computational effort to improve solutions in  $A$  is approximately equally distributed among all regions of the approximation front.

The procedure *destroySol* (cf. Alg. 5) destroys a given solution  $X$ . This is done by removing bundle bids from  $X$  randomly with a destroy rate (removal probability) of  $ds_i$  percent. The destroy rate  $ds_i$  depends on the destroy strategy  $ds$  and the numbers  $fail[X].P$  and  $fail[X].Q$  of failed attempts to improve  $X$  (cf. Alg. 5, remark R1). The destroy probability is the probability by which a bundle bid  $b \in X$  is removed from  $X$ . The function  $rand(1, 100)$  returns a random number between 1 and 100 (inclusively). The procedure *destroySol* returns the resulting solution  $X' \subseteq X$  which is probably infeasible.

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**Algorithm 5:** *destroySol*

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**Input:**  $X, fail\_P, fail\_Q, ds$   
**Output:**  $X' \subseteq X$

$X' \leftarrow X$

R1  $i \leftarrow \min(fail\_P, fail\_Q) \bmod |ds|$

**foreach**  $b \in X^d$  **do**  
    | **if**  $rand(1, 100) \leq ds_i$  **then**  $X' \leftarrow X' \setminus \{b\}$   
**end**

**return**  $X'$

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The destroyed solution  $X'$  is passed to the procedure *repairSol* (cf. Alg. 6). Furthermore, the procedure *repairSol* gets the numbers of failed attempts  $fail.P$  and  $fail.Q$  to improve  $X \supseteq X'$  by using rating function  $P(., X)$  and  $Q(., X)$ , respectively. A new feasible solution is searched for via a single-criterion greedy heuristic. The heuristic chooses among  $P(., X)$  and  $Q(., X)$  that function, which produced less unsuccessful improvement attempts to generate a new non-dominated solution (cf. Alg. 6, remark R1). During the while-loop, the solution  $X'$  is repaired by consecutively adding bids to  $X'$  in a greedy fashion. If the greedy rating function returns  $+\infty$  for a bundle bid, this bid cannot improve the solution and therefore must not be considered in further iterations (cf. Alg. 6, remark R2).

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**Algorithm 6:** *repairSol*

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**Input:** infeasible solution  $X', fail.P, fail.Q$   
**Output:** feasible solution  $X \supset X'$

R1 **if**  $fail.P < fail.Q$  **then**  $g(., .) \leftarrow P(., .)$   
**else**  $g(., .) \leftarrow Q(., .)$

**while**  $X'$  *infeasible* **do**

$z^* \leftarrow \infty$

$b^* \leftarrow \emptyset$

**foreach**  $b \in B \setminus X$  **do**

**if**  $g(b, X') < z^*$  **then**

$z^* \leftarrow g(b, X')$

$b^* \leftarrow b$

**end**

// most greedy bid

R2 **else if**  $g(b, X') = \infty$  **then**  $B \leftarrow B \setminus \{b\}$

$X \leftarrow X \cup \{b^*\}$

**end**

**return**  $X$

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The improvement phase terminates, after reaching a preset time limit.

### 3.3. Note on a Mathheuristic Extension

Buer and Pankratz [14] introduced an exact branch-and-bound method based on the epsilon constraint approach for the 2WDP-SC. This approach, denoted as  $\epsilon LBB$ , was successfully used in Buer and Pankratz [15] to hybridize the

path-relinking phase of a GRASP method for the 2WDP-SC. Obviously, we therefore also tried to further improve the solution quality of PNS by integrating  $\epsilon$ LBB in three ways: 1) hybridizing  $\epsilon$ LBB and *dominanceBasedConstruction* (Alg. 2), 2) hybridizing  $\epsilon$ LBB and *localSearch*-Phase (Alg. 2), and 3) using  $\epsilon$ LBB in both phases. All in all, given the same computing time, the three hybridized approaches led to inferior results compared to PNS. Therefore, we do not pursue this research direction further.

#### 4. Design of computational study

The performance of the proposed heuristic is measured by means of a computational study. This section gives remarks on the test procedure, presents the used benchmark instances, and introduces measures from the literature for the quality of an approximation set.

##### 4.1. General remarks and test procedure

The computational evaluation is done by means of artificial benchmark instances. All algorithms were implemented in Java (JDK 6, Update 23). All tests were executed on the same type of personal computer (CPU Intel core i5-750, four cores a 2.66 GHz). This also includes those heuristics that were published previously (cf. Sect. 5.3), that is, previous computational experiments were repeated if necessary. Up to four independent computational test runs were executed in parallel, however, the implementation of the evaluated heuristics uses no parallelization.

We first evaluate some main design choices of the method PNS. At the same time, we work out reasonable values for the three external parameters of the method PNS. Finally, the new method PNS is compared to three other heuristics from the literature.

##### 4.2. Benchmark instances

The 37 benchmark instances for the 2WDP-SC introduced by Buer and Pankratz [14] are used. These instances take into account some specific features of the transportation scenario at hand. In particular, the instance generation procedure creates bundle bids that satisfy the free disposal assumption. This is important, as this assumption was required to model the 2WDP-SC with set covering constraints (instead of set partitioning constraints).

For seven instances, all Pareto optimal solutions are known. These instances are denoted as *small instances* (instance group S). The small instances feature up to 80 bundle bids. For the remaining thirty instances, the set of Pareto optimal solutions is unknown. These instances are denoted as *large instances*. These instances are divided into different classes by means of different groups classifying instances according to their number of bundles or their number of contracts. There are three groups A, B, and C which contain instances with 500, 1000, and 2000 bundle bids, respectively. The groups a, b, c denote instances with 125, 250, and 500 transport contracts, respectively. Consequently, the class Cb for example contains those instances with 2000 bundle bids and 250 transport contracts. This notation is used in Tab. 4, Tab. 5, and Tab. 6.

##### 4.3. Quality indicators for approximation sets

The assessment of the quality of an approximation set is a nontrivial task. An intensive examination of different approaches is given by Zitzler et al. [21]. The evaluation of algorithms in view of the obtained solution quality is usually more complex in the multiple objective case than in the single objective case. In the single objective case, performance statements are naturally made by comparing the objective function values of solutions generated by different algorithms. However, in multi objective case, approximation sets have to be compared whose fronts cross each other. Given two approximation sets  $A$  and  $B$  with solutions in  $A$  that dominate solutions in  $B$  and the other way round ( $a < b$  and  $a' < b'$  for  $a, a' \in A$  and  $b, b' \in B$ ) makes performance comparisons difficult.

One way to measure approximation set quality is the usage of quality indicators which should narrow down the comparison of two approximation sets to the comparison of two real-valued numbers. Roughly speaking, a quality indicator is a function that assigns to one or more approximation sets a scalar value. This always goes along with a loss of information intrinsic to the approximation sets. Hence, it is advisable to use more than one quality indicator to balance the individual strengths and weaknesses of indicators (which are discussed e.g. by Zitzler et al. [21]). Therefore, we use three different quality indicators for the computational study, the hypervolume indicator, the multiplicative epsilon indicator, and the coverage indicator. Those quality indicators seem to be among the most readily accepted in the literature.



#### 4.3.1. Hypervolume indicator $I_{HV}$

The hypervolume indicator  $I_{HV}(A)$  measures the volume of the objective subspace that is weakly dominated by the solutions of a given approximation set  $A$  and bounded by a reference point  $r$  [22, 23]. The reference point  $r$  has to be weakly dominated by each solution. Higher indicator values imply a better approximation set. Fig. 2 (left) shows three non-dominated solutions  $a^1, a^2, a^3$ . The part of the objective space that is dominated by these solutions and bounded by the reference point  $r$  is shaded in gray. The volume of the gray area is the value of  $I_{HV}(\{a^1, a^2, a^3\})$ . In Fig. 2 (right) the new non-dominated solution  $a^4$  is added and the hypervolume is increased by the volume of the area shaded in dark gray. Apparently, every new non-dominated solution increases the value of  $I_{HV}$ .

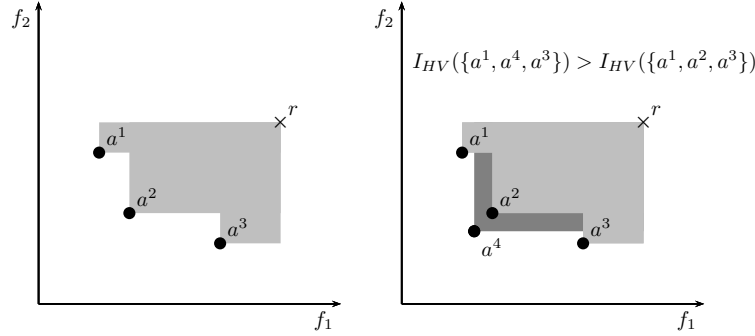


Figure 2: Principle of hypervolume indicator  $I_{HV}$ .

Following earlier studies on the 2WDP-SC [14, 15], the reference point  $r$  is defined as  $r_1 = f_1(B)$  and  $r_2 = 0$ . The values of the objective functions  $f_1$  and  $f_2$  differ in several orders of magnitude (using the benchmark instances of Section 4.2). Therefore they are normalized prior to calculating  $I_{HV}$  according to equation (7).

$$f'_i(X) = \frac{f_i(X) - f_i^{\min}}{r_i - f_i^{\min}} \quad \text{with } i \in \{1, 2\} \quad (7)$$

and  $f_1^{\min} := 0$ ,  $f_2^{\min} := f_2(B) - 1$ .

#### 4.3.2. Epsilon indicator $I_\epsilon$

The multiplicative epsilon indicator  $I_\epsilon(A, B)$  introduced by Zitzler et al. [21, p. 122] compares two approximation sets  $A$  and  $B$  and is based on the epsilon dominance relation  $\leq_\epsilon$ . It is defined as follows:

$$f(a) \leq_\epsilon f(b) \Leftrightarrow \forall i \in \{1, \dots, m\} : f_i(a) \leq \epsilon \cdot f_i(b). \quad (8)$$

$I_\epsilon(A, B)$  is the minimum factor, by which the value of the objective function of each solution in  $B$  has to be multiplied, such that each solution in  $B$  is epsilon dominated by at least one solution in  $A$ .

$$I_\epsilon(A, B) = \inf_{\epsilon \in \mathbb{R}} \{ \forall b \in B, \exists a \in A : f(a) \leq_\epsilon f(b) \}. \quad (9)$$

Lower values of  $I_\epsilon(A, B)$  imply a higher quality of  $A$ . By definition, it holds that  $I_\epsilon(A, B) \geq 1$ . For  $I_\epsilon(A, B) = 1$ , each solution in  $B$  is weakly dominated by a solution in  $A$ . In general,  $I_\epsilon(A, B) \neq I_\epsilon(B, A)$  holds.

$I_\epsilon(A, B)$  is a binary indicator. In case that more than two approximation sets should be compared, a pairwise comparison of the involved approximation sets is required. To simplify the comparison, in this study, we use the unary epsilon indicator [24, S. 12]:

$$I_\epsilon(A) := I_\epsilon(A, A^R). \quad (10)$$

$A^R$  is denoted as reference approximation set.  $A^R$  is the set union of the approximation sets  $A'$  to be compared without any dominated solutions.

#### 4.3.3. Coverage indicator $I_C$

Zitzler and Thiele [22, S. 297] introduced the binary coverage indicator. The coverage indicator  $I_C(A, B)$  indicates the fraction of solutions in the approximation set  $B$ , that are dominated by at least one solution in the approximation set  $A$ .

$$I_C(A, B) = \frac{|\{b | \exists a \in A : f(a) \leq f(b)\}|}{|B|}. \quad (11)$$

In general,  $I_C(A, B) \neq I_C(B, A)$  holds. Higher values of  $I_C(A, B)$  imply a higher quality of  $A$ . The range of values is  $0 \leq I_C(A, B) \leq 1$ , where  $I_C(A, B) = 1$  indicates, that each solution in  $B$  is dominated by at least one solution in  $A$ . Like  $I_\epsilon(A, B)$ ,  $I_C(A, B)$  is again a binary indicator and we only use the unary variant by means of a reference approximation set:  $I_C(A) := I_C(A, A^R)$ .

## 5. Results and discussion

### 5.1. Contribution of two-stage candidate bid selection

We evaluate, whether the quality of the approximation sets generated by the two-stage candidate bid selection procedure is improved in comparison to a traditional single-stage bid selection procedure. For this, only the construction procedure DRC (cf. Alg. 2) is studied. The single-stage selection procedure is realized by setting the number of sectors  $r$  to 1. The two-stage procedure is realized by using multiple sectors ( $r > 1$ ).

We try to receive an impression of the actual size of the candidate list to identify a reasonable number of sectors  $r$ . Each of the 37 instances was solved 500 times by the heuristic DRC (cf. Alg. 2). Immediately prior to each call of the method *sectorCandSelection*( $CL, k, r$ ) in DRC, the size of the candidate list  $CL$  was measured. Tab. 1 shows the aggregated results. The average size of the candidate list  $CL$  grows slightly with increasing numbers of bundle bids per instance. Nevertheless, even for the largest instances with 2000 bids, the median of  $|CL|$  is only 4 and the maximum size is 21. To avoid an insufficient small number of bids per sector, we use three sectors ( $r = 3$ ) in the two-stage bid selection approach.

Table 1: Size of the candidate list  $CL$  during construction phase.

Instance group	Mean	Stand. dev.	Median	Max.
S (<100 bids)	3.00	1.47	3	7
A (500 bids)	4.19	2.22	4	16
B (1000 bids)	4.29	2.51	4	16
C (2000 bids)	4.58	2.91	4	21

Construction of 500 solutions for each instance.

The construction heuristic with a single-stage bid selection is denoted as  $DRC_{r=1}$  and the two-stage bid selection heuristic is denoted as  $DRC_{r=3}$ . In contrast to Alg. 2, both heuristics terminate after 1000 constructed solutions (and  $d^{max} = \infty$ ). For each of the thirty large instances five test runs with different random seeds were performed. A test run is the one-time solution of an instance with both heuristics  $DRC_{r=1}$  and  $DRC_{r=3}$ . The results for the quality indicators  $I_{HV}$ ,  $I_\epsilon$ , and  $I_C$  are shown in Tab. 2. Note that approximation set  $A^R$  was calculated on a per test run basis and not over all five test runs per instance.

Applying the Wilcoxon signed rank test to the results, the null hypothesis ('the quality indicator median values of the different algorithms possess the same probability distribution') can be rejected for each of the three quality indicators. The p-values for  $I_{HV}$ ,  $I_\epsilon$ , and  $I_C$  are  $\leq 0.0001$ ,  $\leq 0.0001$ , and 0.0028, respectively. The results are statistically significant even for very tight levels of significance of one percent or lower. Therefore, it is highly likely, that the observed quality differences of the obtained approximation sets can be attributed to the usage of the two-stage bid selection procedure in the construction phase.

Regarding the median values of the quality indicators  $I_{HV}$  and  $I_\epsilon$ , the two-stage approach  $DRC_{r=3}$  clearly outperforms the one-stage heuristic  $DRC_{r=1}$ . In contrast, the median values of  $I_C$  suggest an opposite interpretation. Looking at the generated approximation sets, the two-stage heuristic seems to discover better extreme solutions than  $DRC_r = 1$ ,

Table 2: One-stage versus two-stage selection of bids from the candidate list with  $\text{DRC}_{r=1}$  and  $\text{DRC}_{r=3}$ .

	$I_{HV}$		$I_\epsilon$		$I_C$	
	$\text{DRC}_{r=1}$	$\text{DRC}_{r=3}$	$\text{DRC}_{r=1}$	$\text{DRC}_{r=3}$	$\text{DRC}_{r=1}$	$\text{DRC}_{r=3}$
$Q_{25}$	0.8815	0.8924	1.09	1.13	0.07	0.05
$Q_{50}$	0.8917	0.8992	3	1.2	0.11	0.09
$Q_{75}$	0.9352	0.9526	13	1.29	0.15	0.13

especially with respect to  $f_1$ . On the other hand,  $\text{DRC}_{r=1}$  seems to generate more and better compromise solutions with balanced  $f_1$  and  $f_2$  values. This might explain the slightly better values of  $I_C$  in Tab. 2 for the algorithm  $\text{DRC}_{r=1}$ . All in all, the two-stage candidate bid selection approach clearly increases the quality of the calculated approximation sets.

### 5.2. Contribution of dynamic destroy rates in improvement phase

The goal of this experiment is twofold. On the one hand, we want to check if the proposed dynamic destroy rates in the solution phase improve approximation set quality. On the other hand, a proper destroy strategy is searched for. A destroy strategy is a sequence  $\langle ds_1, \dots, ds_n \rangle$  of destroy rates. 17 different destroy strategies are compared, Tab. 3 shows the results. Each of the 17 different destroy strategies is used to compute the thirty large instances twice. These 17 strategies include the five non-dynamic strategies  $\langle 3 \rangle$ ,  $\langle 6 \rangle$ ,  $\langle 9 \rangle$ ,  $\langle 12 \rangle$ ,  $\langle 15 \rangle$  with an a priori fixed destroy rate which is unchangeable. We also experimented with larger destroy rates between 20 and 40 percent, however, these seem clearly inferior to the smaller destroy rates shown in Tab. 3. Column two to four of Tab. 3 show the median of the appropriate quality indicator over 60 runs of the destroy strategy. The runtime for each run was fixed to five minutes. The best median values are bold.

Table 3: Results for 17 different destroy strategies for PLNS.

strategy	$I_{HV}$	$I_\epsilon$	$I_C$
$\langle 3, 6, 9 \rangle$	0.9095	<b>1.01</b>	0.01
$\langle 6, 12, 18 \rangle$	0.9093	1.02	0.00
$\langle 9, 18, 27 \rangle$	0.9089	1.03	0.00
$\langle 9, 6, 3 \rangle$	<b>0.9097</b>	1.015	0.01
$\langle 18, 12, 6 \rangle$	0.9096	1.02	0.00
$\langle 27, 18, 9 \rangle$	0.9093	1.03	0.00
$\langle 3 \rangle$	0.9096	<b>1.01</b>	<b>0.07</b>
$\langle 6 \rangle$	0.9096	1.02	0.02
$\langle 9 \rangle$	0.9096	1.02	0.00
$\langle 12 \rangle$	0.9092	1.025	0.00
$\langle 15 \rangle$	0.9092	1.03	0.00
$\langle 5, 15, 7 \rangle$	0.9096	1.02	0.00
$\langle 7, 19, 9 \rangle$	0.9093	1.02	0.00
$\langle 15, 5, 10 \rangle$	0.9094	1.02	0.00
$\langle 19, 7, 14 \rangle$	0.9095	1.02	0.00
$\langle 3, 6, 9, 2, 4 \rangle$	<b>0.9097</b>	<b>1.01</b>	0.05
$\langle 6, 12, 18, 5, 10 \rangle$	0.9096	1.02	0.00

Median values of quality indicators over two runs of each large instance.

The strategies  $\langle 3 \rangle$ , which is static, and  $\langle 3, 6, 9, 2, 4 \rangle$  achieve the best median values for two quality indicators, respectively. We use both strategies to compute each of the large instances five times. Applying the Wilcoxon signed rank test to the results, the null hypothesis ('the quality indicator median values of the different algorithms possess the same probability distribution') can be rejected for two of the three quality indicators on a level of significance of

less than three percent. The p-values for  $I_{HV}$ ,  $I_\epsilon$ , and  $I_C$  are  $\leq 0.0001$ , 0.0216, and 0.4231, respectively. The dynamic strategy clearly outperforms the static strategy by means of the hypervolume indicator while the observed difference by means of the coverage indicator is not significant. We conclude, that the dynamic strategy  $\langle 3, 6, 9, 2, 4 \rangle$  works best. Fig. A.3 in the appendix depicts three runtime distributions which indicate that this strategy usually obtains a given target value faster.

### 5.3. Comparison with other heuristics

To benchmark the new method PNS by means of approximation set quality the three heuristics SPEA2A, PGRASP<sub>P</sub>+HPR, and PGRASP<sub>Q</sub>+HPR are used. The method SPEA2A is based on the *Strength Pareto Evolutionary Algorithm 2* introduced by Zitzler et al. [25]. This generic multi objective genetic algorithm was *adapted* by genetic operators specific to the 2WDP-SC in [14]. In that paper the method was called A<sub>8</sub>. PGRASP<sub>P</sub>+HPR, and PGRASP<sub>Q</sub>+HPR were proposed in [15]. Both methods are multi objective GRASP whose path-relinking phase was hybridized with the exact branch-and-bound method  $\epsilon LBB$  by [14]. Another hybridized heuristic for the 2WDP-SC was discussed in [16] (see also 3.3) which is, however, not included in our comparison, as it does not clearly outperform the mentioned heuristics on the majority of instances.

For the benchmark, the parameters of PNS are set as follows. The number of sections  $r$  are set to 3, the vector destroy probabilities is set to  $(3, 6, 9, 2, 4)$ , and the termination criterion of the construction phase is set to  $d^{max} = 92$ . While the configuration of the first two values were justified in Sections 5.1 and 5.2, the value of the termination criterion  $d^{max}$  was determined as follows: for each large instance, 1000 solutions were generated with DRC (cf. Alg. 2). The experimental distribution of the number of unsuccessful improvement tries was recorded (median 6, mean 20, standard deviation 42) and  $d^{max}$  was set to the value of the ninety-five percent quantile, which is 92.

The runtime of each heuristic was five minutes (300s). All heuristics solved all instances on the same type of computer. Please note, we *do not cite* the computational results of the experiments in [14, 15] but solve all instances again on the same (and faster) computer.

The results for the small instances with known Pareto optimal solution sets are shown in Tab. 4. The two rightmost columns show the Pareto optimal hypervolume values and the cardinality of the reference approximation set  $A^R$  (here, it is identical to the Pareto optimum solution set). These optimal results haven been obtained by the bicriteria branch-and-bound method  $\epsilon LBB$  introduced in [14].

Algorithm PNS is able to solve all seven small instances to Pareto optimality, that is the whole Pareto optimal solution set is found. In contrast, the procedures PGRASP<sub>P</sub>+HPR and PGRASP<sub>Q</sub>+HPR are able to solve six out of seven instances to Pareto optimality. In [15], only four instances could be solved to Pareto optimality. The method SPEA2A is able to find some Pareto optimal solutions for six instances (S1 – S5, S7), but never the complete set.

Table 4: Comparison of solution approaches by means of small instances (instance group S).

Instance	PNS			PGRASP <sub>P</sub> +HPR			PGRASP <sub>Q</sub> +HPR			SPEA2A			$\epsilon LBB^*$	
	$I_{HV}$	$I_\epsilon$	$I_C$	$I_{HV}$	$I_\epsilon$	$I_C$	$I_{HV}$	$I_\epsilon$	$I_C$	$I_{HV}$	$I_\epsilon$	$I_C$	$I_{HV}$	$ A^R $
S1	0.8576	1.00	1.00	0.8576	1.00	1.00	0.8576	1.00	1.00	0.8573	1.03	0.71	0.8576	7
S2	0.6095	1.00	1.00	0.6095	1.00	1.00	0.6095	1.00	1.00	0.6022	1.08	0.45	0.6095	11
S3	0.8169	1.00	1.00	0.8169	1.00	1.00	0.8169	1.00	1.00	0.8125	1.47	0.38	0.8169	13
S4	0.5677	1.00	1.00	0.5677	1.00	1.00	0.5677	1.00	1.00	0.5636	1.41	0.25	0.5677	12
S5	0.8652	1.00	1.00	0.8644	1.01	0.88	0.8652	1.02	0.94	0.8535	2.00	0.29	0.8652	17
S6	0.6988	1.00	1.00	0.6988	1.00	1.00	0.6988	1.00	1.00	0.6879	1.27	0.10	0.6988	10
S7	0.8915	1.00	1.00	0.8915	1.00	1.00	0.8915	1.00	1.00	0.8866	1.66	0.12	0.8915	17

\*The method  $\epsilon LBB$  calculates always Pareto-optimal solutions.

The results for the large instances without known Pareto optimum solution are shown in Tab. 5. This time, the reference approximation set  $A^R$  (cf. two rightmost columns) is generated by merging the approximation sets of PNS, PGRASP<sub>P</sub>+HPR, PGRASP<sub>Q</sub>+HPR, and SPEA2A and removing the dominated solutions. The last five rows of Tab. 5 show the 25 percent quantile, the median, the 75 percent quantile, the mean, and the standard deviation for each heuristic and each quality indicator.

The heuristic PNS finds new best approximation sets in terms of  $I_{HV}$  and  $I_\epsilon$  for all thirty instances. Therefore, PNS clearly outperforms the existing approaches in terms of approximation set quality. Furthermore, from the values

of  $I_C$  follows that the approximation sets computed by PNS are even equal to the reference approximation set  $A^R$  in 28 out of 30 instances. Only for the instances Ba3 and Cc7, the reference approximation set is not solely generated by PNS. Consequently, the solution approach PNS obtains for all three quality indicators the best median indicator values at the same time.

Table 5: Comparison of solution approaches by means of large instances (instance groups A, B, and C).

Instance	PNS			PGRASP <sub>P</sub> +HPR			PGRASP <sub>Q</sub> +HPR			SPEA2A			reference	
	$I_{HV}$	$I_\epsilon$	$I_C$	$I_{HV}$	$I_\epsilon$	$I_C$	$I_{HV}$	$I_\epsilon$	$I_C$	$I_{HV}$	$I_\epsilon$	$I_C$	$I_{HV}$	$ A^R $
Aa1	0.9027	1.00	1.00	0.8996	1.14	0.00	0.8929	1.11	0.00	0.8895	1.33	0.00	0.9027	68
Aa2	0.9132	1.00	1.00	0.9118	1.06	0.00	0.9056	1.09	0.00	0.9016	1.42	0.00	0.9132	43
Aa3	0.9063	1.00	1.00	0.9026	1.04	0.00	0.8996	1.08	0.00	0.8979	1.29	0.00	0.9063	60
Ba1	0.9559	1.00	1.00	0.9521	1.21	0.00	0.9510	1.13	0.00	0.9475	1.43	0.00	0.9559	100
Ba2	0.9596	1.00	1.00	0.9578	1.11	0.00	0.9545	1.14	0.00	0.9502	1.85	0.00	0.9596	80
Ba3	0.9619	1.00	0.99	0.9595	1.22	0.00	0.9557	1.19	0.01	0.9521	1.36	0.00	0.9619	70
Bb1	0.9084	1.00	1.00	0.9050	1.17	0.00	0.9013	1.08	0.00	0.8971	1.34	0.00	0.9084	58
Bb2	0.9070	1.00	1.00	0.9036	1.13	0.00	0.9009	1.07	0.00	0.8990	1.29	0.00	0.9070	65
Bb3	0.9050	1.00	1.00	0.9033	1.14	0.00	0.8984	1.07	0.00	0.8960	1.33	0.00	0.9050	51
Bb4	0.9143	1.00	1.00	0.9071	1.36	0.00	0.9024	2.00	0.00	0.8840	20.00	0.00	0.9143	149
Bb5	0.9071	1.00	1.00	0.9006	1.12	0.00	0.8988	1.10	0.00	0.8913	2.00	0.00	0.9071	108
Bb6	0.9102	1.00	1.00	0.9041	1.24	0.00	0.8993	1.13	0.00	0.8941	1.35	0.00	0.9102	114
Ca1	0.9809	1.00	1.00	0.9798	1.21	0.00	0.9783	1.18	0.00	0.9579	11.00	0.00	0.9809	100
Ca2	0.9825	1.00	1.00	0.9809	1.35	0.00	0.9794	1.22	0.00	0.9793	1.53	0.00	0.9825	85
Ca3	0.9812	1.00	1.00	0.9781	2.00	0.00	0.9786	1.17	0.00	0.9691	5.00	0.00	0.9812	73
Cb1	0.9585	1.00	1.00	0.9560	1.15	0.00	0.9529	1.15	0.00	0.9527	1.27	0.00	0.9585	78
Cb2	0.9589	1.00	1.00	0.9567	1.21	0.00	0.9539	1.13	0.00	0.9527	1.35	0.00	0.9589	50
Cb3	0.9569	1.00	1.00	0.9544	1.09	0.00	0.9530	1.09	0.00	0.9512	1.32	0.00	0.9569	30
Cb4	0.9594	1.00	1.00	0.9546	2.00	0.00	0.9533	1.17	0.00	0.9410	13.00	0.00	0.9594	143
Cb5	0.9621	1.00	1.00	0.9581	1.42	0.00	0.9556	1.19	0.00	0.9495	7.00	0.00	0.9621	119
Cb6	0.9586	1.00	1.00	0.9537	1.33	0.00	0.9524	1.17	0.00	0.9507	1.54	0.00	0.9586	100
Cc1	0.8991	1.00	1.00	0.8914	2.00	0.00	0.8883	1.11	0.00	0.8773	27.00	0.00	0.8991	147
Cc2	0.9083	1.00	1.00	0.8980	3.00	0.00	0.8974	1.15	0.00	0.8894	16.00	0.00	0.9083	164
Cc3	0.9043	1.00	1.00	0.8972	1.30	0.00	0.8944	1.11	0.00	0.8923	1.29	0.00	0.9043	128
Cc4	0.9087	1.00	1.00	0.9059	1.03	0.00	0.9046	1.05	0.00	0.9013	1.20	0.00	0.9087	14
Cc5	0.9014	1.00	1.00	0.8996	1.02	0.00	0.8980	1.04	0.00	0.8972	1.05	0.00	0.9014	8
Cc6	0.8980	1.00	1.00	0.8962	1.02	0.00	0.8949	1.03	0.00	0.8931	1.21	0.00	0.8980	12
Cc7	0.9001	1.00	0.97	0.8940	1.09	0.00	0.8923	1.08	0.03	0.8916	1.23	0.00	0.9001	86
Cc8	0.9042	1.00	1.00	0.8981	1.19	0.00	0.8955	1.09	0.00	0.8957	1.23	0.00	0.9042	65
Cc9	0.9018	1.00	1.00	0.8932	1.11	0.00	0.8922	1.10	0.00	0.8905	1.22	0.00	0.9018	89
$Q_{25}$	0.9045	1.00	1.00	0.8996	1.11	0.00	0.8976	1.08	0.00	0.8925	1.29	0.00	0.9045	59
$Q_{50}$	<b>0.9095</b>	<b>1.00</b>	<b>1.00</b>	0.9055	1.18	0.00	0.9019	1.11	0.00	0.8985	1.35	0.00	<b>0.9095</b>	79
$Q_{75}$	0.9588	1.00	1.00	0.9557	1.32	0.00	0.9532	1.17	0.00	0.9506	1.96	0.00	0.9588	106
$\mu$	0.9292	1.00	1.00	0.9251	1.32	0.00	0.9225	1.15	0.00	0.9178	4.35	0.00	0.9292	82
$\sigma$	0.0303	0.00	0.01	0.0315	0.42	0.00	0.0320	0.17	0.01	0.0315	6.50	0.00	0.0303	41

$Q_{50}$  denotes the median. The best median-values are bold.  $Q_{25}$  and  $Q_{75}$  denote the lower and upper quartile, respectively.  
 $\mu$  and  $\sigma$  denote mean and standard deviation, respectively.

#### 5.4. Runtime behavior

To compare the runtime of the three heuristics from the literature with the new method PNS, a target hypervolume value is defined for each instance. The runtime needed to achieve the target value is measured.

The target value is defined as the lowest  $I_{HV}$  value per instance shown in Tab. 4 and Tab. 5. Therefore, we are sure that each heuristic has been able to reach the target value at least once. The best known hypervolume value seems not to be a qualified target value because this target value will probably not be reached by most heuristics, which limits the value of the experiment. Following, each instance is solved by each heuristic 75 times and the time to target is measured. The total runtime per heuristic was limited to three minutes (180s). Note, if an algorithm could not reach the target value within 180s, then a runtime of 180s is reported anyway. Therefore, an algorithm might appear faster than it actually is. However, this behavior occurred only with the heuristic SPEA2A and never with the heuristic PNS.

The aggregated results are reported in Tab. 6. According to the reported median values for the 37 instances, the new heuristic PNS ranks second. The fastest method is PGRASP<sub>P</sub>+HPR, third place goes to PGRASP<sub>Q</sub>+HPR, and fourth place goes to SPEA2A. The median runtime of the method PGRASP<sub>Q</sub>+HPR for the larger instances is around

135s, which can be explained by a switch from the neighborhood search phase towards the path relinking phase, which is time dependent. Furthermore, although SPEA2A can repeatedly not achieve the target value in the predefined 180s (cf.  $Q_{75}$  in Tab. 6), for some of the larger instances SPEA2A seems competitive (cf.  $Q_{25}$  and  $Q_{50}$ ).

Table 6: Comparison of the runtime (s) of the four heuristics for instance groups S, A, B, and C.

Group	PNS			PGRASP <sub>P</sub> +HPR			PGRASP <sub>Q</sub> +HPR			SPEA2A		
	$Q_{25}$	$Q_{50}$	$Q_{75}$	$Q_{25}$	$Q_{50}$	$Q_{75}$	$Q_{25}$	$Q_{50}$	$Q_{75}$	$Q_{25}$	$Q_{50}$	$Q_{75}$
S	0.03	0.07	0.12	0.02	<b>0.06</b>	0.2	0.06	0.16	0.635	104.16	168.36	182.67
A	2.64	3.66	5.04	0.30	<b>0.58</b>	1.82	88.36	135.05	145.61	183.05	182.94	183.20
B	7.86	11.75	18.48	0.91	<b>1.80</b>	3.89	10.81	135.05	135.19	8.14	97.55	182.95
C	24.07	35.28	52.54	3.50	<b>10.20</b>	36.67	3.74	135.05	137.79	13.61	26.27	182.73
S. A. B. C	3.50	16.65	34.37	0.52	<b>2.39</b>	10.75	1.45	87.66	135.17	14.30	142.33	182.87

To give more insights into the runtime behavior of the heuristics, Fig. A.4(a)-(f) show the experimental runtime distribution for selected instances. The method SPEA2A is missing on some figures, as this procedure is sometimes too slow and the respective curve would lie too distant from the other curves.

## 6. Conclusion

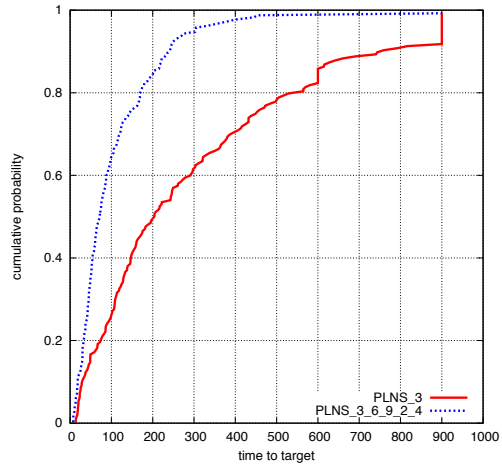
Considering quality aspects during winner determination in a combinatorial reverse auction for transport contracts is of practical importance. In this paper, we studied a bi-objective winner determination problem that is based on the set covering problem and minimizes the total transport costs and the total transport quality simultaneously. To solve this problem, the heuristic PNS was developed. PNS is inspired by the metaheuristics GRASP and LNS. To construct an initial set of non dominated solutions, PNS applies a dominance-based randomized greedy heuristic which uses a two-stage candidate bid selection procedure. The initial solutions are improved by means of a search in large neighborhoods which switches the applied parameters (removal probability of bids and greedy rating function) in a self-adaptive manner. Self-adaptive configurations depend on individual solutions and not on the entire approximation set. PNS was tested by means of 37 benchmark instances. In terms of approximation set quality, PNS outperforms all known heuristics on each of the 37 benchmark instances. Furthermore, PNS is the second fastest method tested. Subject of our future research will be the development of solution approaches for bi-objective winner determination problems which take into account additional business constraints proposed e.g. by Caplice and Sheffi [6].

## Appendix A. Time to target plots

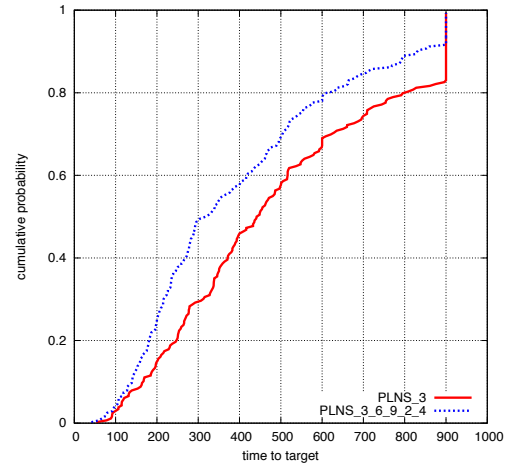
Hoos and Stützle [26] as well as Ribeiro et al. [27] discuss the evaluation of algorithms by runtime distributions. *Time to target* plots were introduced by Feo et al. [28]. To draw the plots presented in this appendix the program of Aiex et al. [29] was used.

## References

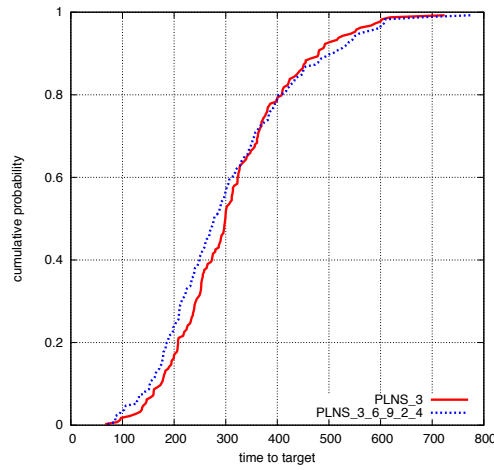
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(a) Instance Aa2, target value  $I_{HV} = 0.9131$

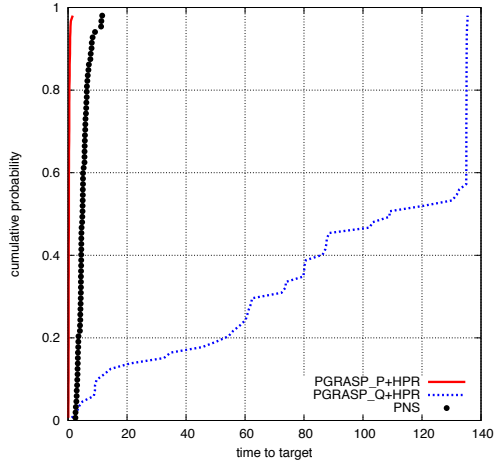


(b) Instance Bb6, target value  $I_{HV} = 0.9097$

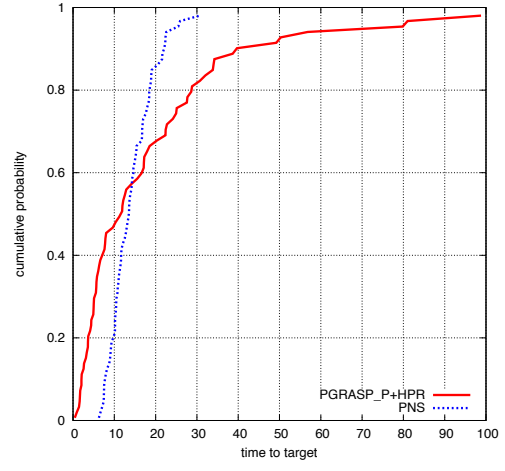


(c) Instance Cc5, target value  $I_{HV} = 0.9046$

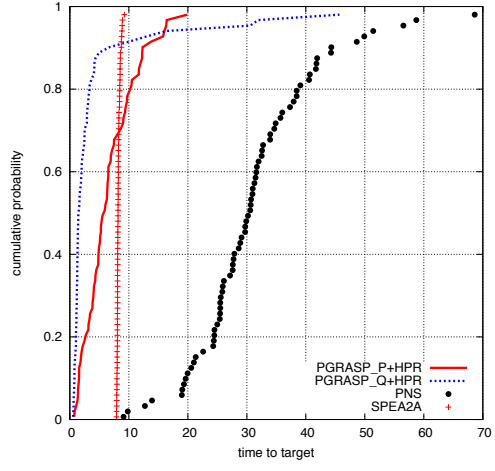
Figure A.3: Empirical runtime distribution of  $\text{PLNS}_{(3)}$  and  $\text{PLNS}_{(3,6,9,2,4)}$  (dotted) determined by 200 runs



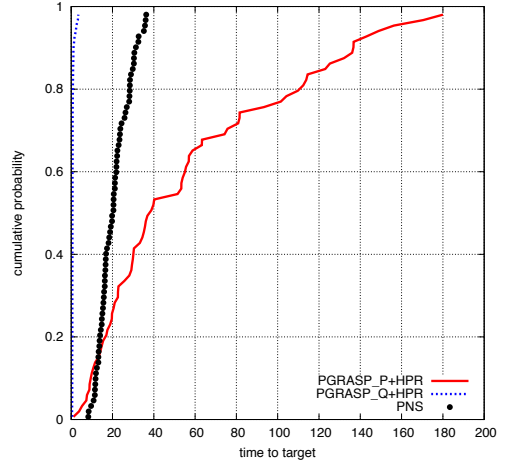
(a) Instance Aa2



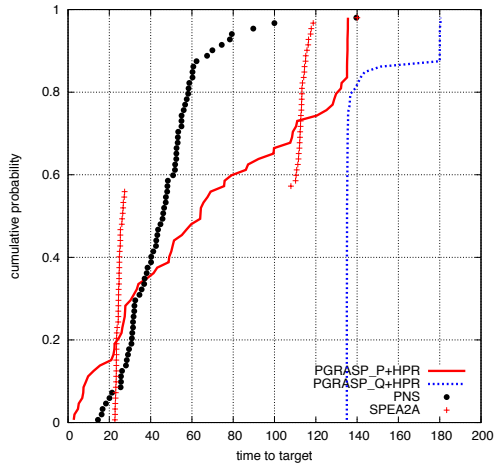
(b) Instance Bb2



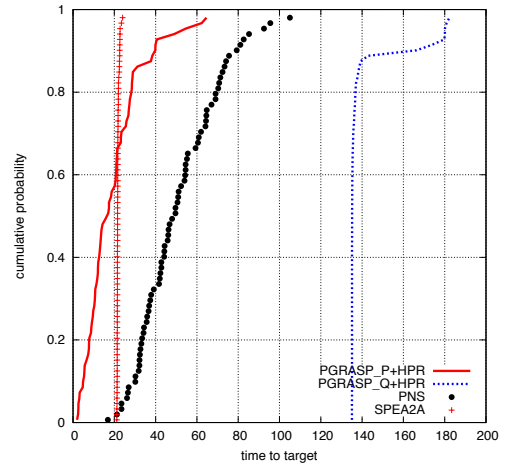
(c) Instance Bb4



(d) Instance Ca3



(e) Instance Cc6



(f) Instance Cc9

Figure A.4: Empirical runtime distribution of PNS compared to benchmark heuristics determined by 75 runs



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